MEAM-620

Advanced Robotics

Roller Racer

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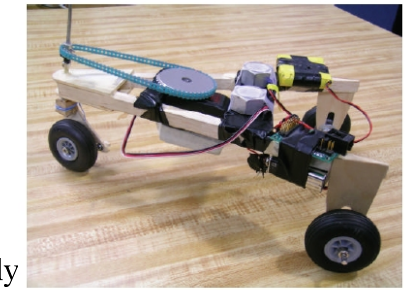
Introduction:

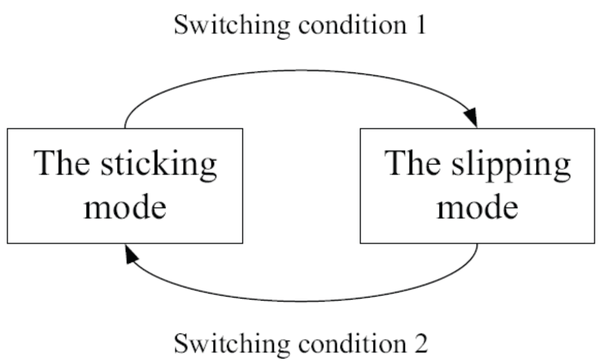


Roller racer is a three wheeled vehicle manually operated by hands. The drive is given to the front wheel and the rear two wheels are stationary. The front wheel is controlled by a sinusoidal control which generates a forward motion of the Roller racer. The overall weight of the Roller racer is concentrated on the rear side so that the rear wheels do not slip or lose ground contact at any point of time. The electronically operated version (and the one which is considered for problem formulation) of Roller racer is as shown in the adjacent figure.

Background:

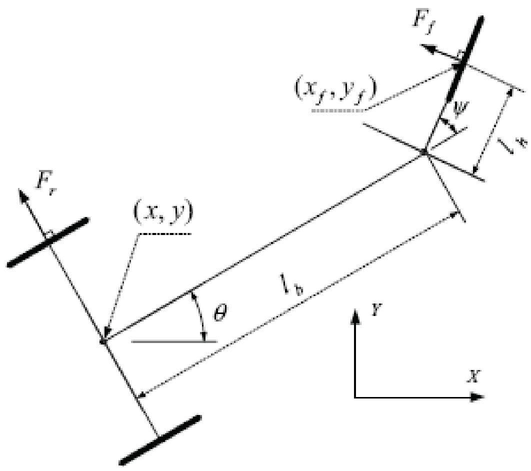
There have been several efforts in the past to develop a mathematical model for a Roller racer. It was initially modeled considering no slipping i.e. infinite friction between wheels and the ground. Mathematically it has been proved that such a system with no slipping has very less or no controllability. Also, such a system is practically impossible since there always occurs a slip when the frictional forces between wheels and the ground exceed kinetic frictional forces. Hence to devise an effective mathematical formulation, a finite friction model was proposed by Peng et al [1] in 'Motion Planning for the Roller Racer with a Sticking/Slipping Switching Model'. Because the friction between the wheels and the ground is considered finite, Roller racer starts slipping once the boundary condition is overcome. It has been shown in the paper that we can achieve better controllability with such assumption. Here, we use the same mathematical model and verify different claims proposed in the paper mentioned above.

Hybrid systems approach:

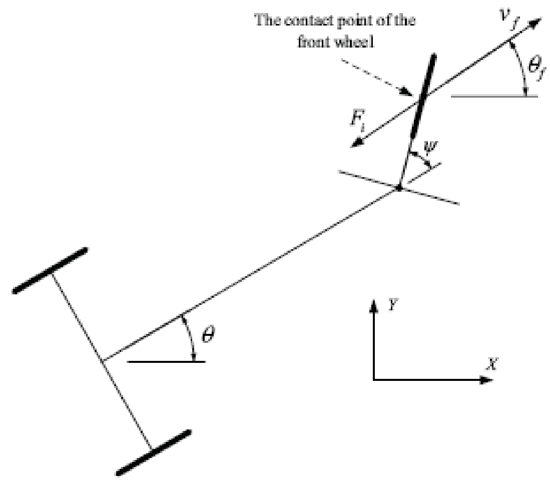


Hybrid systems are those which exhibit both continuous as well as discrete dynamics. The system intermittently jumps between several of its discrete modes, and each of these discrete modes has some continuous dynamics associated with it. In current project, we have considered three discrete modes namely Sticking mode, Slipping mode, and Rest mode. The system autonomously switches between these three modes as and when the continuous dynamics associated with that mode change in a specific manner. One such switching condition between Sticking and Slipping modes is as shown in the figure above. When the continuous dynamics associated with Sticking mode satisfy certain conditions imposed on them to change its state, the system jumps to the slipping mode, and vice versa. Eventually, the system becomes stable at the desired point.

Central Idea:



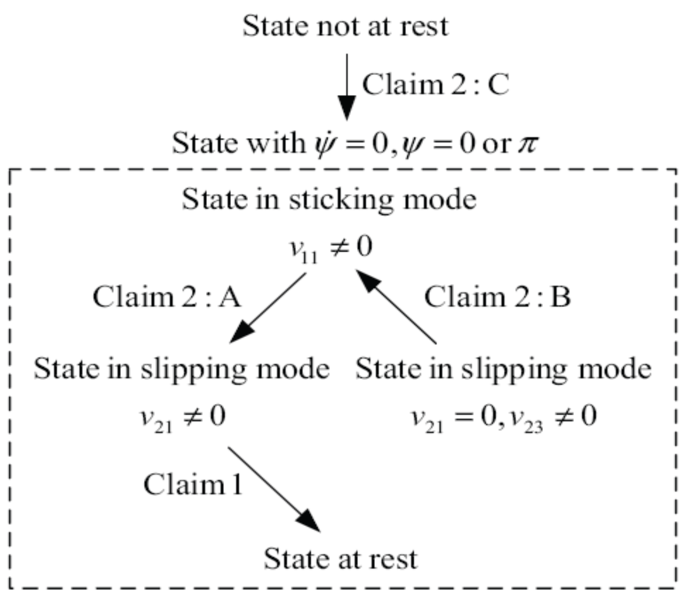
(Sticking mode)



(Slipping mode)

To formulate the mathematical model of the Roller racer, the system is considered as shown in the figure above. It has got two rear wheels and a single front wheel. Θ is the angular displacement of the body and Ψ is angular displacement of the steering handle. Our primary goal is to to stop the vehicle from any non-zero velocities to rest, just by controlling the steering angle Ψ. The secondary goal is to find an optimal control for the steering angle to stop the Roller racer at the desired point in minimum time.

Problem formulation:



As stated earlier, there are three modes of operation of the Roller racer namely Sticking mode, Slipping mode, and Rest mode. The conditions required to be satisfied for the Roller racer to be in any particular mode are shown above in the figure. In addition to that, different claims which can be used to jump between each of those modes as and when required. When the system voids any particular constraint to be in any specific mode, it takes a discrete jump and enters some other mode. So the system keeps on jumping between these modes autonomously until it arrives to the desired mode.

Method description:

Lets divide the overall methodology into two main criteria:

1. Modes
2. Claims

1. **Modes:** These are the desirable states of the Roller racer, achieved because of the autonomous switches during its operation. For any mode, the important elements to be considered are position of the Roller racer, orientation of its body and steering handle, and linear and angular velocities of the Roller racer and its steering handle. The mode-wise dynamics of the system are explained as follows:

   a. Sticking mode:
   Dimension of velocity vector field – 2
   Velocity vectors update according to established mathematical relations
   Position is updated according to the velocity vectors
   At each time instance, it is checked whether system satisfies the conditions required to be in the Sticking mode
   b. Slipping mode:
   Dimension of velocity vector field – 3, one extra dimension due to slipping velocity
   Velocity vectors update according to established mathematical relations
   Position is updated according to the velocity vectors
   At each time instance, it is checked whether system satisfies the conditions required to be in the Slipping mode
   c. Rest mode:
   All the velocities in this mode are zero and the system is at rest.

   The dynamics for all the modes are formulated as ordinary differential equations. To solve these dynamics, function ODE45 in MATLAB was used.

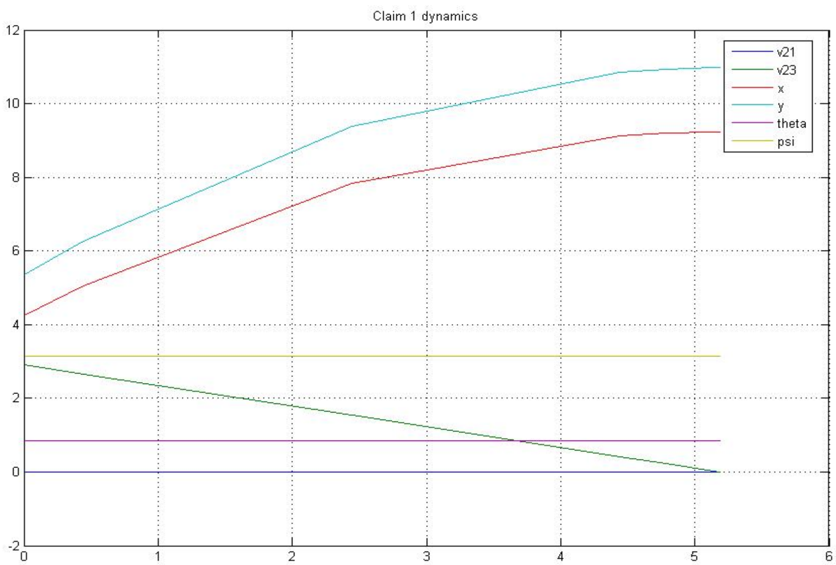
2. **Claims:** These are the mathematical proofs explained in the paper [1] which yield the control to be applied to the Roller racer in order to make it go into the desired state. Several such claims are explained below in short:

   a. Claim 2C:
   This applies control to the state with non-zero velocities to any other state which can be Stick, Slip, or Rest
   b. Claim 2A:
   This claim applies control to the system already in the sticking mode to the slipping mode
   c. Claim 2B:
   This claim applies control to the system in the slipping mode and makes it go into the sticking mode
   d. Claim 1:
   This claim applies control to the system in the slipping mode and takes it to the rest

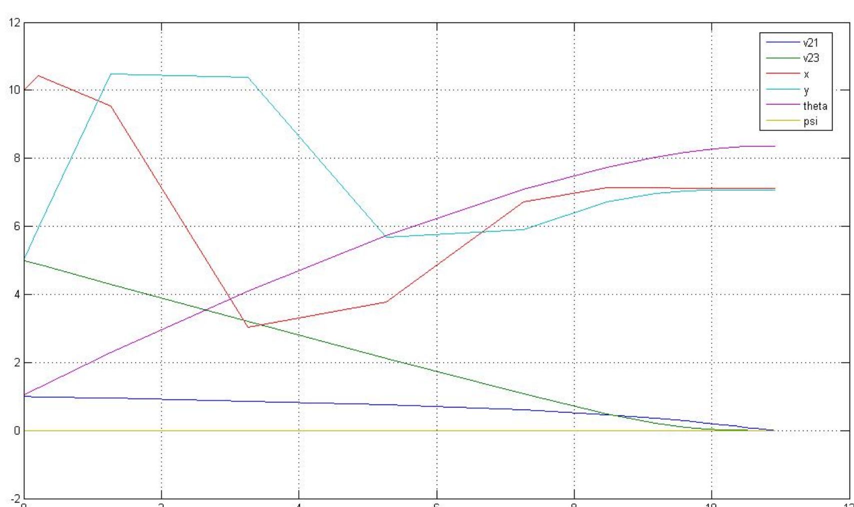
The simulation works as follows. The MATLAB program first asks user to input initial configuration of the Roller racer and its initial velocities. Then according to the conditions required to be satisfied to be in any specific mode, program decides which mode to enter first. Program then keeps on jumping the system between several modes until it reaches to the desired mode.

Simulation and results:
The simulation results for some of the modes are explained below:

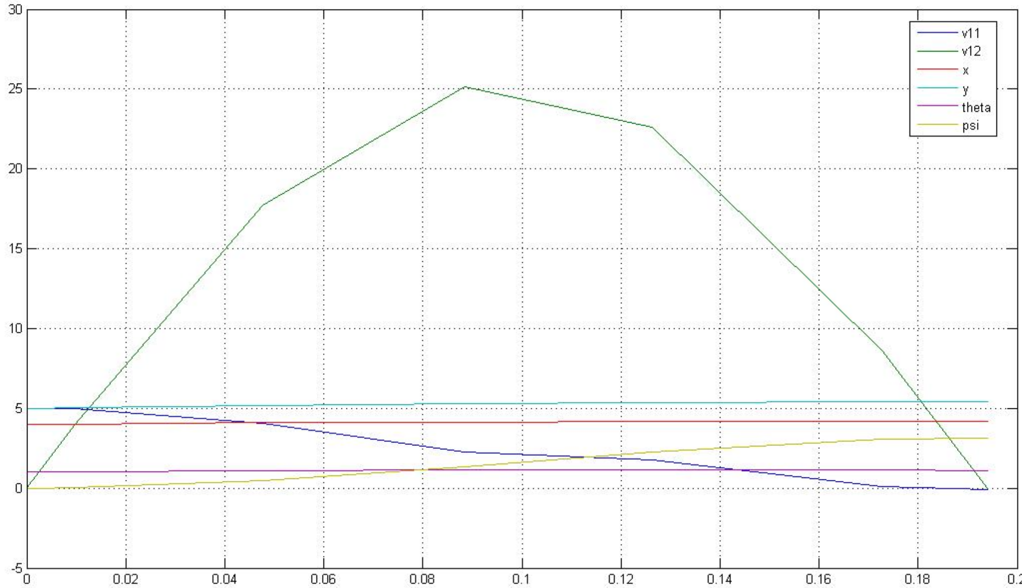
Claim 1 dynamics:



As we know, system should come to a complete halt after it has been controlled by claim 1. If we have a look at the graph, we can see that all the variables representing the position of the Roller racer (x, y, theta, psi) are stabilized after finite time i.e. System is now in the rest mode. Similar results for some different initial configuration of the Roller racer are shown below.



Claim 2A dynamics:



After applying Claim 2A, the system is supposed to go from a sticking mode to slipping mode where steering angle velocity becomes zero. We can observe exactly same dynamics in the figure given above. Here, variable v12 represents the steering angle velocity. After increasing for some period, it decreases to zero and system goes into slipping mode.

Optimal Control for Switched Systems:

Problem statement:

Starting from an initial position, find the optimal control that will stop the Roller racer in desired state within minimum possible time.

Proposed theory:

The basic theory which we intend to use to solve such optimal control problem is the one which is proposed in 'Applications of Numerical Optimal Control to Nonlinear Hybrid Systems' by Shangming Wei et al [2]. It has been observed that there exist numerically sound algorithms for obtaining suboptimal solutions for a hybrid optimal control, for example Bellman inequality and Mixed Index Programming (MIP). The main challenge in designing an optimal control is combinatorial optimization. The above stated methods eventually approach to a solution but there is a great risk of combinatorial explosion. This means there can be infinite cases involved in finding the optimal solution and hence these methods do not scale well. To get over this challenge, an optimal Model Predictive Control (MPC) method is proposed in 'Applications of Numerical Optimal Control to Nonlinear Hybrid Systems'. This method utilizes traditional non-linear programming techniques such as Sequential Quadratic Programming (SQP). SQP dramatically reduces computational complexity over the existing approaches. If the system exhibits autonomous switches, the optimal control problem can be solved by traditional techniques and if it exhibits controlled switches, advanced techniques proposed in [4] can be utilized to break the problem into simpler steps. The general steps involved in formulating the optimum control problem are as follows:

1. Find optimal sequence of control modes
2. Find optimal switching instants
3. Find optimum value for continuous control

Difficulties faced during the execution of project:

1. Interpretation of mathematical relations
2. Numerical integration

   For numerical integration, first we tried a crude method of approximation due to which there were a lot of discrepancies between the expected and the actual answers. But then we learnt and implemented the ODE45 function in MATLAB for the integration and finally got the expected results.

3. Implementing ODE45 function

Conclusions:

1. The mathematical model proposed in [1] has been successfully verified
2. Using finite friction model, we can stop the Roller racer in finite time at desired point
3. Hybrid system approach makes it simpler to model the system
4. Introduction of slip to the front wheel increases controllability of the Roller racer

So far, we have not been successful in implementing the mathematical model proposed in [2]. Currently we are working on the formulation of the mathematical model to achieve the goal mentioned in the problem statement, and we are optimistic in achieving good results in near future. To design the optimal control for the Roller racer, we have gone through several research papers, and we can draw following conclusions based on the experiments and observations mentioned in those references:

1. Traditional computational techniques such as SQP can be effectively used to solve the optimal control problems for an autonomously switching system.
2. If system exhibits a controlled switch, then there exist some advanced techniques [4] which can be used to simplify the complexity involved in the computation of the optimum control problem.
3. Above two techniques can be effectively used to find an optimum time for a Roller racer to go from an initial position to the desired final position.

References:

[1] Motion Planning for the Roller Racer with a Sticking/Slipping Switching Model

Peng Cheng, Emilio Frazzoli, Vijay Kumar

[2] Applications of Numerical Optimal Control to Nonlinear Hybrid Systems

Shangming Wei, Kasemsak Uthaichana, Miloˇs ˇZefran, Raymond A. DeCarlo, Sorin Bengea

[3] RoboTrikke: A Novel Undulatory Locomotion System

Sachin Chitta, Peng Cheng, Emilio Frazzoli, Vijay Kumar

[4] Kinematic Controllability for Decoupled Trajectory Planning in Underactuated Mechanical Systems

Francesco Bullo, Associate Member, IEEE, and Kevin M. Lynch, Member, IEEE