Investigation of Formal Verification for Self-Healing Analog/RF Systems

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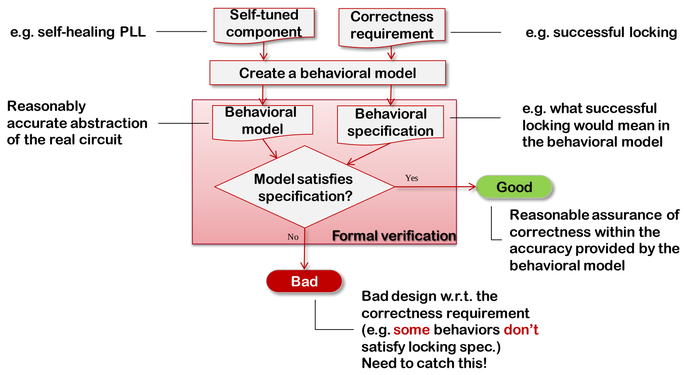
Motivation for formal verification

ANALYSIS TASK	ANALYSIS METHOD
Analysis of a single operating point	**Simulation**
Analyze the correctness of design	Simulate one particular behavior
Analysis with process variations	**Monte Carlo simulation**
Analyze robustness against process variations	Simulate many behaviors
Analysis over complete post-silicon tuning range	**Formal verification?**
Determine whether there are acceptable solutions in the tuning range	State space too large for simulation! Verify all possible behaviors of a reasonably accurate behavioral model

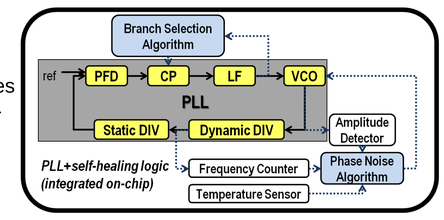
How we can use formal verification

Verification-aided design of self-tuned components

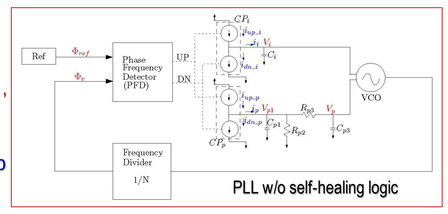


Target application: self-healing PLL

- **Verify locking behavior over**
  - arbitrary initial states
  - range of parameter values
  - with self-healing logic

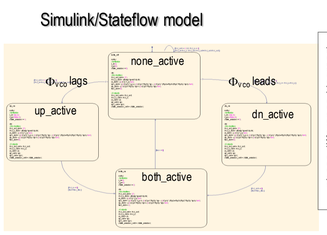


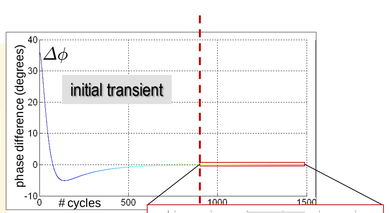
- **Behavioral model**
  - Continuous state variables: Φ_{ref}, Φ_v, V_i, V_{p1}, V_p
  - Discrete switching due to charge pump operation

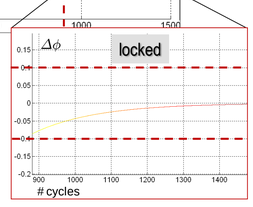


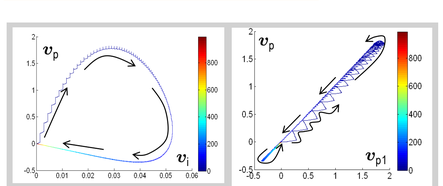
Simulation of the behavioral model

Simulink/Stateflow model





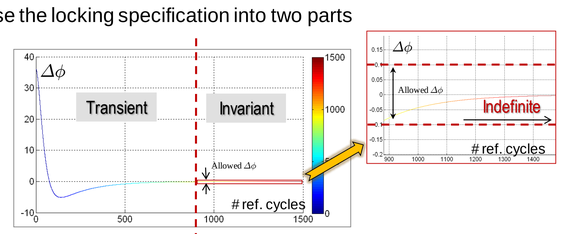




Provides quick insight about the behavior, but can only simulate one behavior at a time. Over a wide range, need to simulate many behaviors one-by-one. This is costly.

Verification approach

Decompose the locking specification into two parts



Transient verification

- Bounded-time verification of whether all behaviors enter the invariant target

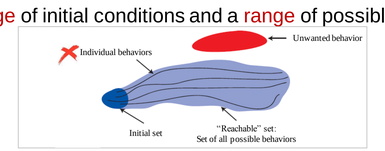
Invariant verification

- Identify regions of state space that guarantee staying in the limit indefinitely
- This becomes a target set for transient verification

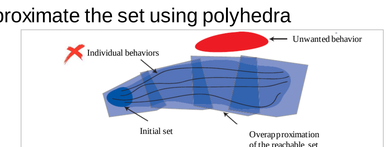
Verification using reachability analysis

General approach

- **Compute the set of all behaviors (not one-by-one)**
  - for a range of initial conditions and a range of possible dynamics



- **If reachable set is hard to compute (typically the case)**
  - over-approximate the set using polyhedra



Challenges in reachability analysis

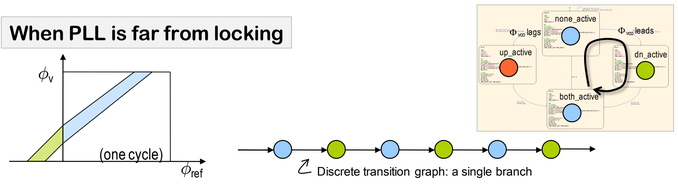
- **Hybrid dynamics**
  - Verification complexity exponential in the number of continuous state variables for polyhedral computations
  - With zonotope (polyhedra with special structure) computations*, there's major speed-up in continuous reachability (cubic complexity); but complexity still exponential for hybrid dynamics
- **Very long transient**
  - Thousands of discrete transitions; over-approximation becomes less accurate with each discrete transition
- **Liveness specification (locking)**
  - Need to verify indefinite (infinite-time) behavior
  - Over-approximation grows with time

* *Antoine Girard,* Reachability of Uncertain Linear Systems Using Zonotopes. HSCC 2005

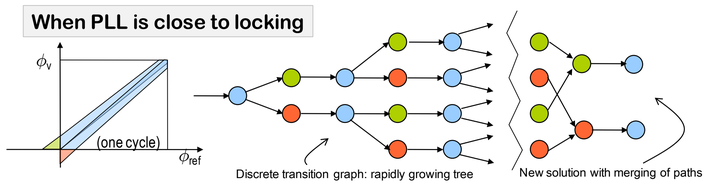
Transient verification using CORA*

Fighting excessive growth of the reachability tree

When PLL is far from locking



When PLL is close to locking

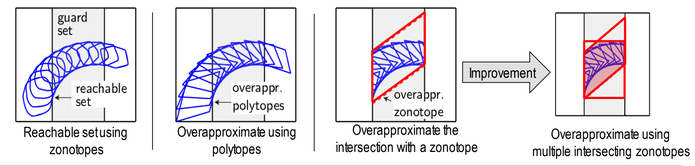


* CORA : COntinuous Reachability Analyzer. *Althoff, M.* Reachability Analysis and its Application to the Safety Assessment of Autonomous Cars, TU München, 2010

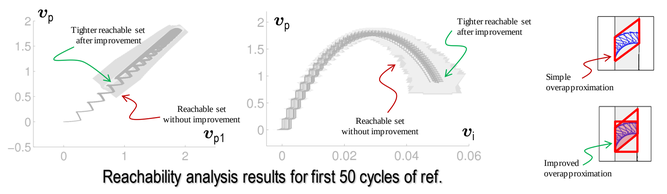
Transient verification using CORA

Making guard set overapproximations tighter

Overapproximation using a single zonotope

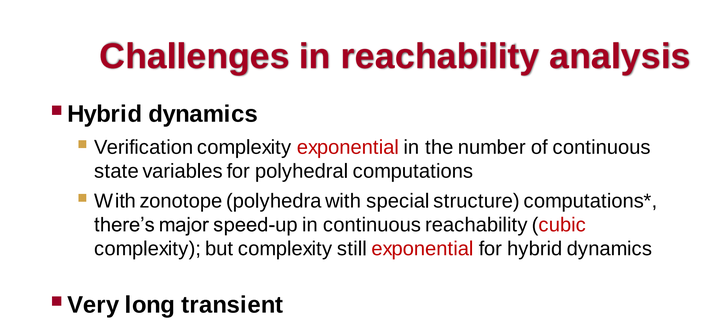


Tighter overapproximation using multiple intersecting zonotopes

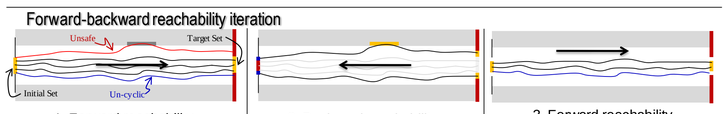


Reachability analysis results for first 50 cycles of ref.

Invariant verification: Forward-backward iteration



Forward-backward reachability iteration



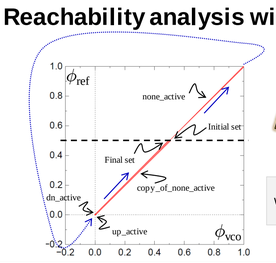
1. Forward reachability
Check for unsafe/uncyclic behavior

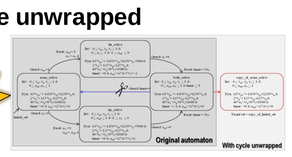
2. Backward reachability
Find unsafe/uncyclic part of initial set

3. Forward reachability
Exclude unsafe/uncyclic initial set, update target set, continue…

Invariant verification using PHAVer*

- **PHAVer (Polyhedral Hybrid Automaton Verifier)**
  - Uses exact rational arithmetic up to arbitrary precision.
  - Supports forward and backward reachability computation.
  - However, needs to overapproximate linear dynamics by (even simpler) piecewise constant bounds on derivatives.
- **Reachability analysis with cycle unwrapped**



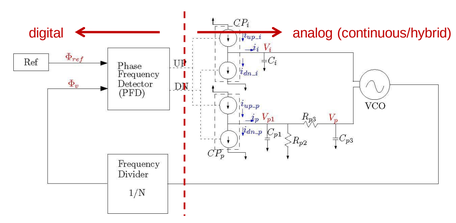


Challenge with PHAVer implementation:
When already locked, charge pumps active for a very short fraction of time. Overapproximation wider than contraction due to charge pump action.

* *Goran Frehse,* PHAVer: algorithmic verification of hybrid systems past HyTech. STTT 10(3): 263-279 (2008)

Next Steps

- **Completion of invariant and transient verification**
- **More detailed model including**
  - Charge pump saturation
  - VCO nonlinearity
- **Compositional verification: digital-analog decoupling**



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